

GEOMETRICAL SCALING AND ITS BREAKING IN HIGH ENERGY COLLISIONS ^a

M. PRASZALOWICZ

*M. Smoluchowski Institute of Physics, Jagiellonian University
4 Reymonta str., 30-059 Krakow, Poland*



We report on recent analyses of different pieces of data, which exhibit Geometrical Scaling (GS) and its breaking. GS is a consequence of the existence of an intermediate energy scale, called saturation momentum, and allows to relate data at different energies, of different systems and also at different multiplicities and/or centralities.

In this talk we give a short overview of searches for the presence of Geometrical Scaling in hadronic collisions. For details we refer the reader to the original publications. Let's start from the formula for the cross-section for inclusive gluon production ¹ in a collision $1 + 2 \rightarrow g + X$:

$$\frac{d\sigma}{dyd^2p_T} = \frac{3\pi}{2p_T^2} \int d^2\vec{k}_T \alpha_s(k_T^2) \varphi_1(x_1, \vec{k}_T^2) \varphi_2(x_2, (\vec{k} - \vec{p})_T^2). \quad (1)$$

Here $\varphi_{1,2}$ are unintegrated gluon densities and $x_{1,2}$ are gluon momenta fractions needed to produce a gluon of transverse momentum p_T and rapidity y :

$$x_{1,2} = e^{\pm y} p_T / \sqrt{s}. \quad (2)$$

Note that unintegrated gluon densities have dimension of area. This is at best seen from the very simple parametrization proposed by Kharzeev and Levin ² or by Golec-Biernat and Wüsthoff ³ in the context of Deep Inelastic Scattering (DIS):

$$\varphi(k_T^2) = S_{\perp} \begin{cases} 1 & \text{for } k_T^2 < Q_s^2 \\ Q_s^2/k_T^2 & \text{for } k_T^2 > Q_s^2 \end{cases} \quad \text{or} \quad \varphi(k_T^2) = S_{\perp} \frac{3}{4\pi^2} \frac{k_T^2}{Q_s^2} \exp(-k_T^2/Q_s^2). \quad (3)$$

Here S_{\perp} is the transverse size given by inelastic cross-section (or its part) for the minimum bias inclusive multiplicity or in the case of DIS $S_{\perp} = \sigma_0$ is the dipole-proton cross-section for large dipoles. Another feature of the unintegrated glue (3) is the fact that φ depends on the ratio

^aPresented at the 49-th Rencontres de Moriond, March 22-29, 2014, La Thuille, Italy.

$k_T^2/Q_s^2(x)$ rather than on k_T^2 and x separately. This is called Geometrical Scaling⁴ and has been for the first time proposed in the context of DIS. Here

$$Q_s^2(x) = Q_0^2(x/x_0)^{-\lambda} = Q_0^2(e^{\pm y} p_T/W)^{-\lambda} \quad (4)$$

is the saturation scale. Taking^b $x_0 = 10^{-3}$ we have $W = \sqrt{s} \times 10^{-3}$ in formula (4).

Assuming particles 1 and 2 to be identical and $y \sim 0$ (central rapidity) which corresponds to $x_1 \simeq x_2$ (denoted in the following as x) and suppressing α_s we arrive at:

$$\frac{d\sigma}{dy d^2p_T} = S_\perp^2 \mathcal{F}(\tau) \quad \text{or} \quad \frac{1}{S_\perp} \frac{dN}{dy d^2p_T} = \mathcal{F}(\tau) \quad (5)$$

where $\tau = p_T^2/Q_s^2(x)$ is scaling variable and dN/dy stands for multiplicity density. Eq.(5) implies that particle spectra at different energies should coincide if plotted in terms τ . In other words they exhibit GS⁵.

We can integrate now (5) over d^2p_T using

$$dp_T^2 = \frac{2Q_0^2}{2+\lambda} \left(W^2/Q_0^2\right)^{\frac{\lambda}{2+\lambda}} \tau^{-\frac{\lambda}{2+\lambda}} d\tau$$

arriving at

$$\frac{dN}{dy} = S_\perp \int \mathcal{F}(\tau) d^2p_T = S_\perp \bar{Q}_s^2 \frac{2\pi}{2+\lambda} \int \mathcal{F}(\tau) \tau^{-\frac{\lambda}{2+\lambda}} d\tau = \frac{1}{\kappa} S_\perp \bar{Q}_s^2 \quad (6)$$

where $1/\kappa$ is a universal, energy independent integral of \mathcal{F} , and

$$\bar{Q}_s^2 = Q_0^2 \left(W^2/Q_0^2\right)^{\frac{\lambda}{2+\lambda}} \quad (7)$$

is an *average* saturation scale, which can be thought of as a solution of the equation

$$Q_s^2(\bar{Q}_s^2/W^2) = \bar{Q}_s^2.$$

It follows that

$$\bar{Q}_s^2 = \frac{\kappa}{S_\perp} \frac{dN}{dy}. \quad (8)$$

Equation (8) means that the average saturation scale is proportional to the gluon density per unit transverse area. One should keep in mind the distinction between saturation scales (4) and (8), since they are interchangeably used in the literature. The theory behind gluon saturation (for a review see Refs. [6,7] and references therein) is Color Glass Condensate^{8,9,10}.

The existence of GS in pp collisions as given by eq.(5) has been indeed observed in the data⁵ and reported at Moriond 2012¹¹. An efficient way to study GS is to form ratios $R_{W_1, W_2}(\tau) = dN/dy dp_T|_{W_1}(\tau) / dN/dy dp_T|_{W_2}(\tau)$ which, according to (5) should be equal 1 over wide range of τ ¹². This requirement allows to find the optimal value of λ which in the case of the LHC data is equal to 0.27, which is a bit smaller than in DIS¹³. It has been shown¹³ that in DIS GS extends up to rather large $x_{\max} \approx 0.08$.

Surprisingly GS scaling works also for the p_T spectra in heavy ion collisions at RHIC energies¹². In the case of heavy ions the saturation momentum scales as $Q_{As}^2 = A^{1/3} Q_s^2$ and the scaling variable is therefore $\tau_A = p_T^2/Q_{As}^2$. This is illustrated in Fig. 1 where charged particles spectra in AuAu and CaCa collisions as measured by PHOBOS are plotted in terms of p_T and $\sqrt{\tau_A}$. Recently GS for the photons produced in different systems (AA, dA and pp), at different energies and at different centralities (*i.e.* at different S_\perp) has been reported¹⁴.

For $y > 0$ two Bjorken x 's (2) can be quite different: $x_1 > x_2$. Therefore by increasing y one can eventually reach $x_1 > x_{\max}$ and violation of GS is expected. To show this¹⁵ we have used pp

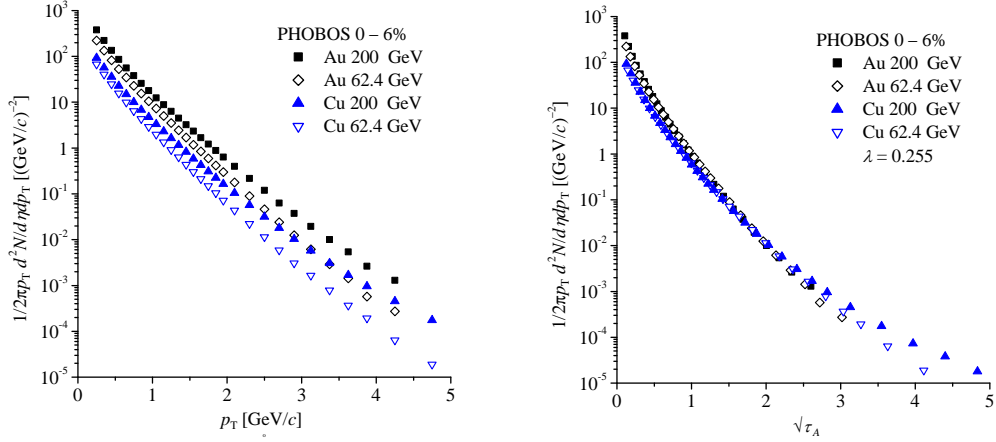


Figure 1 – Transverse momentum spectra measured by PHOBOS in Au-Au and Cu-Cu most central collisions as functions of p_T (right) and $\sqrt{\tau_A}$ (left) (from Ref.[¹²])

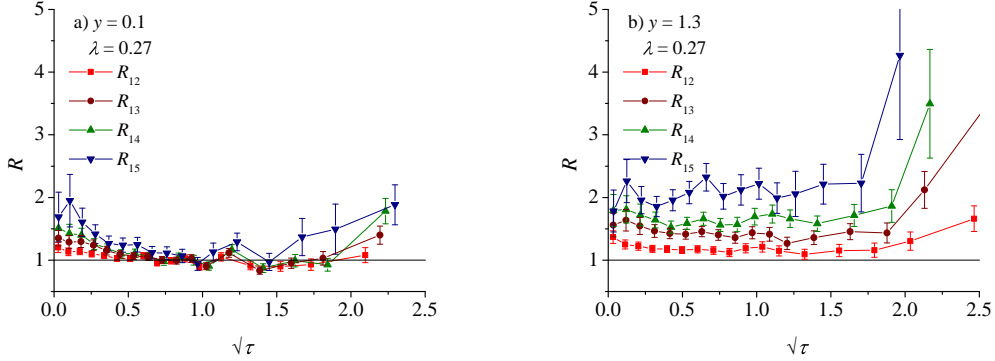


Figure 2 – Ratios R_{1k} as functions of $\sqrt{\tau}$ for $\lambda = 0.27$ and for different rapidities a) $y = 0.7$ and b) $y = 1.3$. With increase of rapidity, gradual closure of the GS region can be seen (from Ref.[¹⁵]).

data from NA61/SHINE experiment¹⁶ which measured particle spectra at different rapidities $y = 0.1 - 3.5$ and at 5 scattering energies $W_{1,\dots,5} = 17.28, 12.36, 8.77, 7.75,$ and 6.28 GeV.

In Fig. 2.a we plot ratios $R_{1i} = R_{W_1, W_i}$ for π^- spectra in rapidity region $y = 0.1$ for $\lambda = 0.27$. Here the GS window extends down to the smallest energy because x_{\max} is as large as 0.08 . Nevertheless one can see that the quality of GS is the worst for the smallest energy W_5 . By increasing y some points fall outside the GS region due to the fact that $x_1 \geq x_{\max}$, and finally for $y \geq 1.7$ geometrical scaling is no longer seen. This is shown in Fig. 2.b.

In a situation where two (or more) external energy scales are present, like p_T and particle mass m (for identified particles), one can form two independent ratios with Q_s what implies violation or at least modification of GS. We have argued that in the case of identified particles GS is still present¹⁷ if another scaling variable is used in which p_T is replaced by $\tilde{m}_T = m_T - m = \sqrt{m_T^2 + p_T^2} - m$. This scaling variable is connected with the fact that accurate fits are obtained by means of Tsallis-like parametrization^{18,19,20} where particle multiplicity distribution takes the following form (see *e.g.* Ref.[21]):

$$\frac{1}{p_T} \frac{d^2N}{dy dp_T} = C \frac{dN}{dy} \left[1 + \frac{m_T - m}{nT} \right]^{-n}. \quad (9)$$

^bThe precise value of Q_0 and x_0 is not important in the following. Only the value of exponent λ will be determined.

Coefficient C ensures proper normalization of (9). Here n and T are free fit parameters that depend on particle species. Formula (9) admits GS solution¹⁷, provided that n is a constant (with possible corrections that would allow for the energy dependence of n seen in the data) and $T \sim \bar{Q}_s$ of eq.(7) which has a power-like energy dependence.

In summary we can say that by now the existence of the saturation scale is undoubtedly well established. Geometrical Scaling follows as a natural consequence. One can use GS to relate different pieces of data with an accuracy much higher than originally expected. New results from the LHC at higher energies will be important for further studies of the details GS and of the underlying theory of dense gluonic system.

Acknowledgements

The author would like to thank the organizers E. Augé and B. Pietrzyk. This work was supported by the Polish NCN grant 2011/01/B/ST2/00492.

References

1. L. V. Gribov, E. M. Levin and M. G. Ryskin, *Phys. Lett. B* **100**, 173 (1981).
2. D. Kharzeev and E. Levin, *Phys. Lett. B* **523**, 79 (2001).
3. K. J. Golec-Biernat and M. Wüsthoff, *Phys. Rev. D* **59**, 014017 (1998) and *Phys. Rev. D* **60**, 114023 (1999).
4. A. M. Stasto, K. J. Golec-Biernat and J. Kwiecinski, *Phys. Rev. Lett.* **86** (2001) 596.
5. L. McLerran and M. Praszalowicz, *Acta Phys. Pol. B* **41** (2010) 1917 and *Acta Phys. Pol. B* **42** (2011) 99.
6. A. H. Mueller, *Parton Saturation: An Overview*, arXiv:hep-ph/0111244.
7. L. McLerran, *Acta Phys. Pol. B* **41**, 2799 (2010).
8. L. V. Gribov, E. M. Levin, and M. G. Ryskin, *Phys. Rept.* **100**, 1 (1983).
9. A. H. Mueller, and J-W. Qiu, *Nucl. Phys. B* **268**, 427 (1986); A. H. Mueller, *Nucl. Phys. B* **558**, 285 (1999).
10. L. D. McLerran, and R. Venugopalan, *Phys. Rev. D* **49**, 2233 (1994) and *Phys. Rev. D* **49**, 3352 (1994) and *Phys. Rev. D* **50**, 2225 (1994).
11. M. Praszalowicz, arXiv:1205.4538 [hep-ph].
12. M. Praszalowicz, *Acta Phys. Pol. B* **42**, 1557 (2011).
13. M. Praszalowicz and T. Stebel, *JHEP* **1303**, 090 (2013) and *JHEP* **1304**, 169 (2013) .
14. C. Klein-Bsing and L. McLerran, arXiv:1403.1174 [nucl-th].
15. M. Praszalowicz, *Phys. Rev. D* **87** (2013) 071502(R).
16. N. Abgrall *et al.* [NA61/SHINE Collaboration], *Report from the NA61/SHINE experiment at the CERN SPS* CERN-SPSC-2012-029, SPSC-SR-107;
A. Aduszkiewicz, Ph.D. Thesis in prepartation, University of Warsaw, 2013;
Sz. Pulawski, talk at 9th Polish Workshop on Relativistic Heavy-Ion Collisions, Kraków, November 2012 and private communication.
17. M. Praszalowicz, *Phys. Lett. B* **727**, 461 (2013).
18. C. Tsallis, *J. Stat. Phys.* **52**,479 (1988); T. S. Biró, G. Purcsel, and K. Ürmösy, *Eur. Phys. J. A* **40**, 325 (2009).
19. M. Rybczynski, Z. Wlodarczyk and G. Wilk, *J. Phys. G* **39**, 095004 (2012).
20. J. Cleymans, G. I. Lykasov, A. S. Parvan, A. S. Sorin, O. V. Teryaev and D. Worku, *Phys. Lett. B* **723**, 351 (2013).
21. S. Chatrchyan *et al.* [CMS Collaboration], *Eur. Phys. J. C* **72**, 2164 (2012).